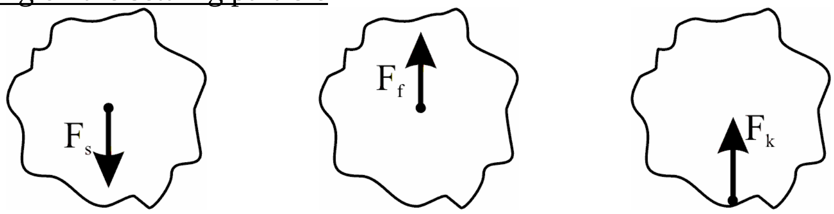
SEDIMENTATION

Forces acting on the settling particle



gravity force   buoyancy force   drag force

In case of floating: their sum is zero.

$$F_s = F_f + F_k$$

$$V \cdot \rho_p \cdot g = V \cdot \rho \cdot g + f \cdot A \cdot \frac{u^2 \cdot \rho}{2}$$

where ρ_p density of particle
ρ density of fluid
A cross section perpendicular to the direction of motion

Rearrangement of the equation

Assuming spherical particle of diameter d, rearrange the equation, and define F(d) and F(u) as:

$$F(d) = \left(f \cdot Re^2\right)^{\frac{1}{3}} = \left[\frac{4}{3} \cdot g \cdot \frac{(\rho_p - \rho) \cdot \rho}{\eta^2}\right]^{\frac{1}{3}} \cdot d_p = B \cdot d_p$$

$$F(u) = \left(\frac{Re}{f}\right)^{\frac{1}{3}} = \frac{u}{B \cdot \nu} = \frac{u \cdot \rho}{B \cdot \eta}$$

Generalized settling plot

Y=F(u) against X=F(d)
Stokes: laminar region
Newton: turbulent region, rarely used because of the very high impulse
Approximation of the Newton region:

$$Y = \sqrt{\frac{X}{0.44}}$$

Exact formula for the Stokes region:

$$Y = \frac{X^2}{24}$$

Problem 1

Sandy sludge is settled in a chanel of base area 1 m × 25 m. Sand density is 2800 kg/m^3, water density is 1100 kg/m^3, its viscosity is 10^{-3} Pas.

a) What is its capacity if the diameter of smallest particle to be settled is $2\cdot10^{-2}$ mm?
b) 40 m^3/min of the same sludge is to be treated in another settler of base area 10 m^2. What diameter particles will be settled?
c) In the case of b), i.e. 40 m^3/min of the same sludge, how many trays are to be applied to settle out particles with diameter 150 μm?

Solution:

$A = 1\text{ m} \times 25\text{ m} = 25\text{ m}^2$
$\rho_p = 2800\text{ kg/m}^3$
$\rho_k = 1100\text{ kg/m}^3$
$\eta_k = 10^{-3}\text{ Pas}$
$d_p = 2\cdot10^{-2}\text{ mm} = 2\cdot10^{-5}\text{ m}$
$\dot{V} = ?$

a) What is its capacity if the diameter of smallest particle to be settled is $2\cdot10^{-2}$ mm?

Capacity of the settler means the maximum flow rate acceptable.
First compute B.

$$B = \left[\frac{4}{3}\cdot g\cdot\frac{(\rho_p-\rho)\cdot\rho}{\eta^2}\right]^{\frac{1}{3}} = \left[\frac{4}{3}\cdot 9.81\frac{m}{s^2}\cdot\frac{\left(2800\frac{kg}{m^3}-1100\frac{kg}{m^3}\right)\cdot 1100\frac{kg}{m^3}}{(10^{-3}\text{Pas})^2}\right]^{\frac{1}{3}} = 2.9\cdot10^4\frac{1}{m}$$

Then, since velocity is to be calculated from known diameter, calculate F(d):

$$X = F(d) = B\cdot d_p = 2.9\cdot10^4\frac{1}{m}\cdot 2\cdot10^{-5}m = 0.58$$

This falls to Stokes regime (according to the plot), thus Y can be calculated with a formula:

$$Y = F(u) = \frac{X^2}{24} = \frac{0.58^2}{24} = 0.014$$

$$F(u) = \frac{u\cdot\rho}{B\cdot\eta}$$

$$u = \frac{F(u)\cdot B\cdot\eta}{\rho} = \frac{0.014\cdot 2.9\cdot10^4\frac{1}{m}\cdot10^{-3}\text{Pas}}{1100\frac{kg}{m^3}} = 3.7\cdot10^{-4}\frac{m}{s}$$

Settling capacity depends on the base area and the settling velocity only.

$$\dot{V} = A\cdot u = 25m^2\cdot 3.7\cdot10^{-4}\frac{m}{s} = 9.26\cdot10^{-3}\frac{m^3}{s} = 33.32\frac{m^3}{h}$$

b) 40 m^3/min of the same sludge is to be treated in another settler of base area 10 m^2. What diameter particles will be settled?

$A' = 10\text{ m}^2$
$\dot{V}' = 40\text{ m}^3/\text{min}$

Settling velocity (its minimum) can be calculated from the given capacity.

$$\dot{V}' = A' \cdot u'$$

$$u' = \frac{\dot{V}'}{A'} = \frac{40\,\frac{m^3}{min}}{10 m^2 \cdot 60\,\frac{min}{h}} = 6.67 \cdot 10^{-2}\,\frac{m}{s}$$

Now unknown diameter is to computed from know velocity. B is the same as earlier because the material is the same.

$$Y' = F(u)' = \frac{u' \cdot \rho}{B \cdot \eta} = \frac{6.67 \cdot 10^{-2}\,\frac{m}{s} \cdot 1100\,\frac{kg}{m^3}}{2.9 \cdot 10^4\,\frac{1}{m} \cdot 10^{-3}\,Pas} = 2.53$$

This value fall to the transient region, thus X' is read from the plot.

$$X' = F(d)' = 13$$

$$d_p' = \frac{F(d)'}{B} = \frac{13}{2.9 \cdot 10^4\,\frac{1}{m}} = 4.48 \cdot 10^{-4}\,m = 0.448 mm$$

Thus, particles with diameter at least as large as 0.448 mm will be settled.

c) In the case of b), i.e. 40 m^3/min of the same sludge, how many trays are to be applied to settle out particles with diameter 150 μm?

A' = 10 m^2

$\dot{V}'$ = 40 m^3/min

d_p'' = 150 μm = $1.5 \cdot 10^{-4}$ m

n = ?

Velocity is to be computed from known diameter.

$$X'' = F(d)'' = B \cdot d'' = 2.9 \cdot 10^4\,\frac{1}{m} \cdot 1.5 \cdot 10^{-4}\,m = 4.35$$

This value fall to the transient region, thus Y" is read from the plot.

$$Y'' = F(v)'' = 0.6$$

$$F(v)'' = \frac{u'' \cdot \rho}{B \cdot \eta}$$

$$u'' = \frac{F(u)'' \cdot B \cdot \eta}{\rho} = \frac{0.6 \cdot 2.9 \cdot 10^4\,\frac{1}{m} \cdot 10^{-3}\,Pas}{1100\,\frac{kg}{m^3}} = 1.58 \cdot 10^{-2}\,\frac{m}{s}$$

How many trays?

$$\dot{V}' = n \cdot A' \cdot u''$$

$$n = \frac{\dot{V}'}{A' \cdot u''} = \frac{40\,\frac{m^3}{min}}{10 m^2 \cdot 1.58 \cdot 10^{-2}\,\frac{m}{s} \cdot 60\,\frac{s}{min}} = 4.22$$

Thus, at least 5 trays are needed. 5 is more than 4.22 but no problem if even smaller particles are also settled out.

Problem 2

After grinding of coal and mullock, a fraction of particle scale 1.2 mm to 1.5 mm have been separated by classifying the particles with sieve.

a) What water velocity must be maintained in a siphon separator in order to separate the two materials?
b) Due to an operation failure, smaller particles of both kinds has got to the system. What is the minimum particle size at which the separation can be performed?

$\eta_{water} = 0.95$ mPas $\qquad \rho_{water} = 1020$ kg/m^3 $\qquad \rho_{coal} = 1200$ kg/m^3
$\rho_{mullock} = 2500$ kg/m^3

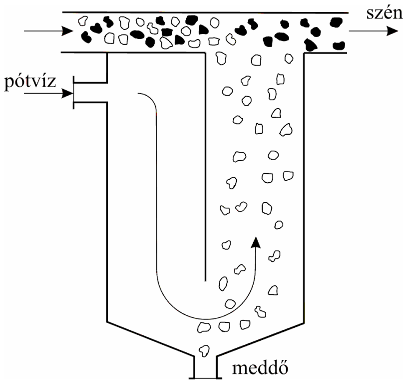


Figure: pótvíz:water supplement, szén: coal, meddő: mullock

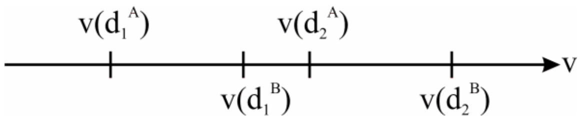
Solution

a) What water velocity must be maintained in a siphon separator in order to separate the two materials?

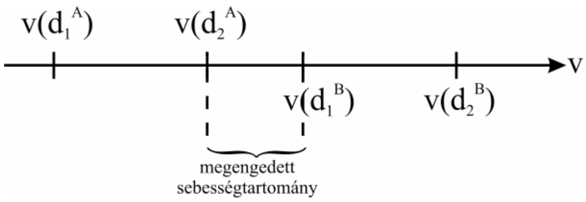
Solution path

Velocity relations

If $d_1 < d_2$ and $\rho_A < \rho_B$ then



Condition of separation



megengedett sebességtartomány: acceptable (feasible) range of velocity

Coal is lighter, teherfore it goes through whereas the heavier mullock settles out. Our target is to settle out even the smallest particles of the heavy mullock and, in the same time, to prevent even the largest particles of the lighter coal from settling out.

$$\left.\begin{array}{l}\rho_{p,smaller} \xrightarrow{d_{p,larger}} u_{min}\\ \rho_{p,larger} \xrightarrow{d_{p,smaller}} u_{max}\end{array}\right\} \rightarrow u_{min} < u < u_{max}$$

Coal

Calculate B_{coal}

$$B_{coal} = \left[\frac{4}{3}\cdot g\cdot\frac{(\rho_{coal}-\rho_{water})\cdot\rho_{water}}{\eta_{water}^2}\right]^{\frac{1}{3}} = \left[\frac{4}{3}\cdot 9.81\frac{m}{s^2}\cdot\frac{\left(1200\frac{kg}{m^3}-1020\frac{kg}{m^3}\right)\cdot 1020\frac{kg}{m^3}}{\left(0.95\cdot 10^{-3}\,Pas\right)^2}\right]^{\frac{1}{3}}$$

$$B_{coal} = 1.39\cdot 10^4\,\frac{1}{m}$$

diameter → velocity:

$$X_{coal} = F(d)_{coal} = B_{coal}\cdot d_{p,larger} = 1.39\cdot 10^4\,\frac{1}{m}\cdot 1.5\cdot 10^{-3}\,m = 20.79$$

Transient region, plot:

$$Y_{coal} = F(u)_{coal} = 4.2$$

$$F(u)_{coal} = \frac{u_{min}\cdot\rho_{water}}{B_{coal}\cdot\eta_{water}}$$

$$u_{min} = \frac{F(u)_{coal}\cdot B_{coal}\cdot\eta_{water}}{\rho_{water}} = \frac{4.2\cdot 1.39\cdot 10^4\,\frac{1}{m}\cdot 0.95\cdot 10^{-3}\,Pas}{1020\,\frac{kg}{m^3}} = 0.054\,\frac{m}{s}$$

Mullock

Calculate $B_{mullock}$.

$$B_{mullock} = \left[\frac{4}{3}\cdot g\cdot\frac{(\rho_{mullock}-\rho_{water})\cdot\rho_{water}}{\eta_{water}^2}\right]^{\frac{1}{3}} = \left[\frac{4}{3}\cdot 9.81\frac{m}{s^2}\cdot\frac{\left(2500\frac{kg}{m^3}-1020\frac{kg}{m^3}\right)\cdot 1020\frac{kg}{m^3}}{\left(0.95\cdot 10^{-3}\,Pas\right)^2}\right]^{\frac{1}{3}}$$

$$B_{mullock} = 2.8\cdot 10^4\,\frac{1}{m}$$

diameter → velocity

$$X_{mullock} = F(d)_{mullock} = B_{mullock}\cdot d_{p,smaller} = 2.8\cdot 10^4\,\frac{1}{m}\cdot 1.2\cdot 10^{-3}\,m = 33.56$$

transient region, plot.

$$Y_{mullock} = F(u)_{mullock} = 6.5$$

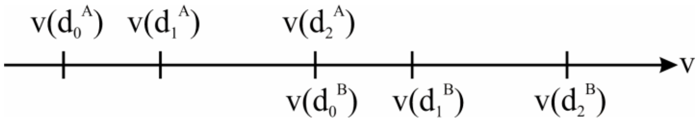
$$F(u)_{mullock} = \frac{u_{max}\cdot\rho_{water}}{B_{mullock}\cdot\eta_{water}}$$

$$u_{max} = \frac{F(u)_{mullock}\cdot B_{mullock}\cdot\eta_{water}}{\rho_{water}} = \frac{6.5\cdot 2.8\cdot 10^4\,\frac{1}{m}\cdot 0.95\cdot 10^{-3}\,Pas}{1020\,\frac{kg}{m^3}} = 0.170\,\frac{m}{s}$$

Thus, velocity of the water supplement must be kept in the range of 0.054 m/s < u < 0.170 m/s.

b) Due to an operation failure, smaller particles of both kinds has got to the system. What is the minimum particle size at which the separation can be performed?

If rather small particles with diameter d_0 are present then the following extreme situation may happen:



Minimum water velocity was determined in problem a) by the settling velocity of the larger particle of the lighter material (coal). At a velocity smaller than this minimum, particles of the lighter material are also settled out.
Now the question is what diameter particles of the heavier material (mullock) will be settled out at the earlier minimum velocity.

$$u = u_{min} = 0.054\frac{m}{s}$$

$d_{p,mullock} = ?$

Diameter is to be computed to known velocity. $B_{mullock}$ is the same as earlier.

$$Y'_{mullock} = F(v)'_{mullock} = \frac{u \cdot \rho_{water}}{B_{mullock} \cdot \eta_{water}} = \frac{0.054\frac{m}{s} \cdot 1020\frac{kg}{m^3}}{2.8 \cdot 10^4 \frac{1}{m} \cdot 0.95 \cdot 10^{-3}\,Pas} = 2.07$$

transient region, plot.

$$X'_{mullock} = F(d)'_{mullock} = 10.5$$

$$d_{p,mullock} = \frac{F(d)'_{mullock}}{B_{mullock}} = \frac{10.5}{2.8 \cdot 10^4 \frac{1}{m}} = 3.75 \cdot 10^{-4}\,m = 0.375\,mm$$

Thus, settling velocity of mullock particles of diameter 0.375 mm equals the settling velocity of the largest coal particles. If such or smaller mullock particles get in the siphon separator then the separation becomes infeasible.

Problem 3

Maximum steam velociy, determined experimentally, is $u_N = 0.5$ m/s at an evaporation process <u>at normal boiling point</u>. Cross section of the vapor channel is 2 m^2, and the pressure in the evaporator is 26664 Pa.
What is the mass flow rate of the steam (water vapor) leaving the evaporator?

Material data at the given and normal pressures:

	26664 Pa	101325 Pa
t_{bp} [°C]	66.5	100
ρ [kg/m^3]	0.1720	0.5977
η [Pas]	$1.065\cdot10^{-5}$	$1.20\cdot10^{-5}$

Water: $\rho_p = \rho_{water} = 1000$ kg/m^3.

Note:
Solvent is removed from a solution in an evaporation process. Solvent vapor is removed and more concentrated solution remains in the vessel. During the boiling and the emergence of vapor, however, liquid droplets are also dragged by the vapor flow and carried away.

For answering the question given in the problem, we assume that the ratio of droplets of different scales are the same in the two cases if the droplet sizes are equal.

<u>Solution</u>

<u>Data:</u>

$v_N = 0{,}5$ m/s
$A = 2$ m^2

Solution path

Acceptable droplet size carried over by the vapor are the same at the two pressures.

$$u_N \xrightarrow{B} F(u) \xrightarrow{\text{Plot}} F(d) \xrightarrow{B} \underline{\underline{d_p}} \xrightarrow{B'} F(d)' \xrightarrow{\text{Plot}} F(u)' \xrightarrow{B'} u' \to \dot{m}'$$

Atmospheric (normal) pressure
Calculate B.

$$B = \left[\frac{4}{3}\cdot g\cdot\frac{(\rho-\rho)\cdot\rho}{\eta^2}\right]^{\frac{1}{3}} = \left[\frac{4}{3}\cdot 9.81\frac{m}{s^2}\cdot\frac{\left(1000\frac{kg}{m^3}-0.5977\frac{kg}{m^3}\right)\cdot 0.5977\frac{kg}{m^3}}{\left(1.20\cdot10^{-5}\,Pas\right)^2}\right]^{\frac{1}{3}} = 3.79\cdot10^4\,\frac{1}{m}$$

velocity → diameter

$$Y = F(u) = \frac{v_N \cdot \rho}{B \cdot \eta} = \frac{0.5\frac{m}{s} \cdot 0.5977\frac{kg}{m^3}}{3.79 \cdot 10^4 \frac{1}{m} \cdot 1.20 \cdot 10^{-5}\,Pas} = 0.658$$

transient region, plot.

$$X = F(d) = 4.5$$

$$d_p = \frac{F(d)}{B} = \frac{4.5}{3.79 \cdot 10^4 \frac{1}{m}} = 1.19 \cdot 10^{-4}\,m$$

Thus, maximum acceptable droplet size is $1.19 \cdot 10^{-4}$ m.

Lower pressure

First calculate B'.

$$B' = \left[\frac{4}{3} \cdot g \cdot \frac{(\rho_p - \rho') \cdot \rho'}{\eta'^2}\right]^{\frac{1}{3}} = \left[\frac{4}{3} \cdot 9.81\frac{m}{s^2} \cdot \frac{\left(1000\frac{kg}{m^3} - 0.172\frac{kg}{m^3}\right) \cdot 0.172\frac{kg}{m^3}}{\left(1.065 \cdot 10^{-5}\,Pas\right)^2}\right]^{\frac{1}{3}} = 2.71 \cdot 10^4 \frac{1}{m}$$

diameter → velocity

$$X' = F(d)' = B' \cdot d_p = 2.71 \cdot 10^4 \frac{1}{m} \cdot 1.19 \cdot 10^{-4}\,m = 3.22$$

transient region, plot.

$$Y' = F(u)' = 0.375$$

$$F(u)' = \frac{u' \cdot \rho}{B' \cdot \eta}$$

$$u' = \frac{F(u)' \cdot B' \cdot \eta' u}{\rho' u} = \frac{0.375 \cdot 2.71 \cdot 10^4 \frac{1}{m} \cdot 1.065 \cdot 10^{-5}\,Pas}{0.172\frac{kg}{m^3}} = 0.63\frac{m}{s}$$

Volumetric flow rate

$$\dot{V}' = u' \cdot A = 0.63\frac{m}{s} \cdot 2m^2 = 1.26\frac{m^3}{s}$$

Mass flow rate

$$\dot{m}' = \dot{V}' \cdot \rho' = 1.26\frac{m^3}{s} \cdot 0.172\frac{kg}{m^3} = 0.216\frac{kg}{s} = 779\frac{kg}{h}$$

Thus, the vapor velocity must not exceed 779 kg/h. At a higher velocity larger droplets would be carried on.

Note: Mass flow rate at atmospheric pressure is

$$\dot{m} = \dot{V} \cdot \rho = v_N \cdot A \cdot \rho = 0.5\frac{m}{s} \cdot 2m^2 \cdot 0.5977\frac{kg}{m^3} = 0.5977\frac{kg}{s} = 2152\frac{kg}{h}$$

Problem 4

Sandy sludg is to be settled in a channel of base area 2 m × 4.5 m and 2 m depth.

$\rho_p = 2800$ kg/m^3 $\rho = 1000$ kg/m^3 $\eta = 10^{-3}$ Pas

a) In case of flow rate 6 m^3/min, how many trays are to be installed to settle out sand particles larger than 50μm? In what region is the settling process performed?
b) If the trays are installed as determined in problem a/ and then the flow rate is doubled then what will be minimum diameter of the settled partices? In what region is the settling process performed?

Data:

$A = 2\ \text{m} \times 4.5\ \text{m} = 9\ \text{m}^2$
$H = 2$ m
$\rho_p = 2800$ kg/m^3
$\rho = 1000$ kg/m^3
$\eta = 10^{-3}$ Pas

a) In case of flow rate 6 m^3/min, how many trays are to be installed to settle out sand particles larger than 50μm? In what region is the settling process performed?

$\dot{V} = 6$ m^3/min = 0.1 m^3/s
$d_p = 50$ μm = $5\cdot10^{-5}$ m

$$B = \left[\frac{4}{3}\cdot g\cdot\frac{(\rho_p-\rho)\cdot\rho}{\eta^2}\right]^{\frac{1}{3}} = \left[\frac{4}{3}\cdot 9.81\frac{\text{m}}{\text{s}^2}\cdot\frac{\left(2800\frac{\text{kg}}{\text{m}^3}-1000\frac{\text{kg}}{\text{m}^3}\right)\cdot 1000\frac{\text{kg}}{\text{m}^3}}{\left(10^{-3}\,\text{Pas}\right)^2}\right]^{\frac{1}{3}} = 2.87\cdot10^4\,\frac{1}{\text{m}}$$

diameter → velocity:

$$X = F(d) = B\cdot d_p = 2.87\cdot10^4\,\frac{1}{\text{m}}\cdot 5\cdot10^{-5}\,\text{m} = 1.433$$

Stokes region, formula.

$$Y = F(u) = \frac{X^2}{24} = \frac{1.433^2}{24} = 0.0856$$

$$F(u) = \frac{u\cdot\rho}{B\cdot\eta}$$

$$u = \frac{F(u)\cdot B\cdot\eta}{\rho} = \frac{0.0856\cdot 2.87\cdot10^4\,\frac{1}{\text{m}}\cdot10^{-3}\,\text{Pas}}{1000\,\frac{\text{kg}}{\text{m}^3}} = 2.46\cdot10^{-3}\,\frac{\text{m}}{\text{s}}$$

The needed number of trays, including the bottom of the channel:

$$\dot{V} = n\cdot A\cdot u$$

$$n = \frac{\dot{V}}{A\cdot u} = \frac{0.1\frac{\text{m}^3}{\text{s}}}{9\text{m}^2\cdot 2.54\cdot10^{-3}\,\frac{\text{m}}{\text{s}}} = 4.52 \rightarrow 5$$ (Only 4 trays are needed to be installed over the bottom of the channel.)

b) If the trays are installed as determined in problem a/ and then the flow rate is doubled then what will be minimum diameter of the settled partices? In what region is the settling process performed?

The total area is changed:

$$A' = n \cdot A = 5 \cdot 9m^2 = 45m^2$$

The new flow rate:

$$\dot{V}' = 2 \cdot \dot{V} = 0.2 \frac{m^3}{s}$$

Settling velocity determined from the flow rate:

$$\dot{V}' = A' \cdot u'$$

$$u' = \frac{\dot{V}'}{A'} = \frac{0.2 \frac{m^3}{s}}{45m^2} = 4.44 \cdot 10^{-3} \frac{m}{s}$$

velocity → diameter , B does not change, it depends on the materials only.

$$Y' = F(u)' = \frac{u' \cdot \rho}{B \cdot \eta} = \frac{4.44 \cdot 10^{-3} \frac{m}{s} \cdot 1000 \frac{kg}{m^3}}{2.87 \cdot 10^4 \frac{1}{m} \cdot 10^{-3} Pas} = 0.155$$

Stokes region, formula.

$$Y' = \frac{X^2}{24} = \frac{F(d)'^2}{24}$$

$$F(d)' = \sqrt{24 \cdot Y'} = \sqrt{24 \cdot 0.155} = 1.93$$

$$d_p' = \frac{F(d)'}{B} = \frac{1,93}{2.87 \cdot 10^4 \frac{1}{m}} = 6.72 \cdot 10^{-5} m = 67.2 \mu m$$

Thus, sand particles larger than 67.2 μm will be settled out.